

SUMMER – 2018 EXAMINATION

Subject Name: Applied Mathematics

Model Answer

Subject Code: **22201**

Important Instructions to Examiners:

- 1) The answers should be examined by key words and not as word-to-word as given in the model answer scheme.
- 2) The model answer and the answer written by candidate may vary but the examiner may try to assess the understanding level of the candidate.
- 3) The language errors such as grammatical, spelling errors should not be given more importance (Not applicable for subject English and Communication Skills).
- 4) While assessing figures, examiner may give credit for principal components indicated in the figure. The figures drawn by candidate and model answer may vary. The examiner may give credit for any equivalent figure drawn.
- 5) Credits may be given step wise for numerical problems. In some cases, the assumed constant values may vary and there may be some difference in the candidate's answer and model answer.
- 6) In case of some questions credit may be given by judgement on part of examiner of relevant answer based on candidate's understanding.
- 7) For programming language papers, credit may be given to any other program based on equivalent concept.

Q. No.	Sub Q. N.	Answer	Marking Scheme
1.		Attempt any FIVE of following:	10
	a)	If $f(x) = x^4 - 2x + 7$, find $f(0) + f(2)$	02
	Ans	$f(x) = x^4 - 2x + 7$ $\therefore f(0) = (0)^4 - 2(0) + 7 = 7$ $\therefore f(2) = (2)^4 - 2(2) + 7 = 19$ $\therefore f(0) + f(2) = 7 + 19$ $\therefore f(0) + f(2) = 26$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	b)	State whether the function $f(x) = \frac{e^x + e^{-x}}{2}$ is odd or even.	02
	Ans	$f(x) = \frac{e^x + e^{-x}}{2}$ $\therefore f(-x) = \frac{e^{-x} + e^{-(-x)}}{2}$ $= \frac{e^{-x} + e^x}{2}$ $= f(x)$ \therefore function is even.	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	c)	If $y = \log(x^2 + 2x + 5)$ then find $\frac{dy}{dx}$	02

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1.	c)	$y = \log(x^2 + 2x + 5)$	
	Ans	$\therefore \frac{dy}{dx} = \frac{1}{x^2 + 2x + 5} \cdot \frac{d}{dx}(x^2 + 2x + 5)$ $= \frac{1}{x^2 + 2x + 5} \cdot (2x + 2) = \frac{2(x+1)}{x^2 + 2x + 5}$	1 1
	d)	Evaluate $\int \frac{1 - \cos 2x}{1 + \cos 2x} dx$	02
	Ans	$\int \frac{1 - \cos 2x}{1 + \cos 2x} dx$ $= \int \frac{2 \sin^2 x}{2 \cos^2 x} dx$ $= \int \tan^2 x dx$ $= \int (\sec^2 x - 1) dx$ $= \tan x - x + c$	½ ½ ½ ½
e)	Evaluate $\int \frac{1}{2x+5} dx$	02	
Ans	$\int \frac{1}{2x+5} dx = \frac{1}{2} [\log(2x+5)] + c$ <p>OR</p> $\int \frac{1}{2x+5} dx = \frac{1}{2} \int \frac{1}{x + \frac{5}{2}} dx$ $= \frac{1}{2} \left[\log \left(x + \frac{5}{2} \right) \right] + c$	2 2	
f)	Find the area under the parabola $y^2 = 4x$, bounded by the lines $x = 0, y = 0, x = 4$	02	
Ans	$\text{Area } A = \int_a^b y dx$ $= \int_0^4 2\sqrt{x} dx$	½	

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1.	f)	$\text{Area } A = \int_0^4 2x^{1/2} dx$ $= \left[2 \frac{x^{3/2}}{3/2} \right]_0^4$ $= \left[\frac{4}{3} x^{3/2} \right]_0^4$ $= \frac{4}{3} [4^{3/2} - 0]$ $= 10.667$	<p>1/2</p> <p>1/2</p> <p>1/2</p>
	g)	State the trapezoidal rule of numerical integration.	02
	Ans	<p>Trapezoidal rule</p> $\int_a^b f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$ <p>where $h = \frac{b-a}{n}$</p>	2
2.		<p>Attempt any THREE of the following:</p>	12
	a)	If $x^y = e^{x-y}$ then prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$	04
	Ans	$x^y = e^{x-y}$ $\log x^y = \log e^{x-y}$ $\therefore y \log x = (x - y) \log e$ $\therefore y \log x = x - y$ $y \log x + y = x$ $y(\log x + 1) = x$ $\therefore y = \frac{x}{1 + \log x}$ $\therefore \frac{dy}{dx} = \frac{(1 + \log x) \frac{d(x)}{dx} - x \frac{d(1 + \log x)}{dx}}{(1 + \log x)^2}$	<p>1/2</p> <p>1/2</p> <p>1</p>

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2.	a)	$\therefore \frac{dy}{dx} = \frac{(1 + \log x) - x \left(\frac{1}{x}\right)}{(1 + \log x)^2}$ $\therefore \frac{dy}{dx} = \frac{1 + \log x - 1}{(1 + \log x)^2}$ $\therefore \frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$	1 ½ ½
	b)	If $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$, then find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$	04
	Ans	$x = a(\theta - \sin \theta)$ $y = a(1 - \cos \theta)$	
		$\therefore \frac{dx}{d\theta} = a(1 - \cos \theta)$ $\therefore \frac{dy}{d\theta} = a \sin \theta$	1+1
		$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	
		$\therefore \frac{dy}{dx} = \frac{a \sin \theta}{a(1 - \cos \theta)} = \frac{\sin \theta}{(1 - \cos \theta)}$ OR $\frac{dy}{dx} = \frac{\sin \theta}{(1 - \cos \theta)} = \frac{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}}{2 \sin^2 \frac{\theta}{2}} = \cot \frac{\theta}{2}$	1
		at $\theta = \frac{\pi}{4}$	
		$\therefore \frac{dy}{dx} = \frac{\sin \frac{\pi}{4}}{\left(1 - \cos \frac{\pi}{4}\right)} = \frac{\frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}}$	
		$\therefore \frac{dy}{dx} = \frac{1}{\sqrt{2} - 1}$ or 2.414 OR $\frac{dy}{dx} = \cot \frac{\pi}{4} = \cot \frac{\pi}{8} = 2.414$	1
	c)	Find maximum and minimum value of $y = x^3 - 18x^2 + 96x$	04
Ans	Let $y = x^3 - 18x^2 + 96x$ $\therefore \frac{dy}{dx} = 3x^2 - 36x + 96$ $\therefore \frac{d^2y}{dx^2} = 6x - 36$ Consider $\frac{dy}{dx} = 0$	½ ½	

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2.	c)	$3x^2 - 36x + 96 = 0$	1/2	
		$x^2 - 12x + 32 = 0$		
		$\therefore x = 8 \text{ or } x = 4$	1/2	
		at $x = 8$		
		$\frac{d^2y}{dx^2} = 6(8) - 36 = 12 < 0$	1/2	
			$\therefore y \text{ is minimum at } x = 8$	
			$y_{\min} = (8)^3 - 18(8)^2 + 96(8)$	
			$= 128$	1/2
			at $x = 4$	
			$\frac{d^2y}{dx^2} = 6(4) - 36 = -12 < 0$	1/2
		$\therefore y \text{ is maximum at } x = 4$		
		$y_{\max} = (4)^3 - 18(4)^2 + 96(4)$		
		$= 160$	1/2	
	d)	Find radius of curvature of the curve $y = x^3$ at $(2, 8)$	04	
	Ans	$y = x^3$		
		$\therefore \frac{dy}{dx} = 3x^2$	1/2	
		$\therefore \frac{d^2y}{dx^2} = 6x$	1/2	
		at $(2, 8)$		
		$\frac{dy}{dx} = 3(2)^2 = 12$	1/2	
		$\frac{d^2y}{dx^2} = 6(2) = 12$	1/2	
		$\therefore \text{Radius of curvature is } \rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$		
		$\therefore \rho = \frac{\left[1 + (12)^2\right]^{\frac{3}{2}}}{12}$	1	
		$\therefore \rho = 145.50$	1	

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3.		Attempt any THREE of the following:	12
	a)	Find $\frac{dy}{dx}$ if $y = x^x + (\sin x)^x$	04
	Ans	Let $u = x^x$ $\therefore \log u = \log x^x$ $\log u = x \log x$ $\frac{1}{u} \frac{du}{dx} = x \frac{1}{x} + \log x (1)$ $\therefore \frac{du}{dx} = u(1 + \log x)$ $\therefore \frac{du}{dx} = x^x (1 + \log x)$ Let $v = (\sin x)^x$ $\therefore \log v = \log (\sin x)^x$ $\log v = x \log (\sin x)$ $\frac{1}{v} \frac{dv}{dx} = x \frac{1}{\sin x} \cos x + \log (\sin x) (1)$ $\frac{1}{v} \frac{dv}{dx} = x \cot x + \log (\sin x)$ $\frac{dv}{dx} = v(x \cot x + \log (\sin x))$ $\frac{dv}{dx} = (\sin x)^x (x \cot x + \log (\sin x))$ $\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $\therefore \frac{dy}{dx} = x^x (1 + \log x) + (\sin x)^x (x \cot x + \log (\sin x))$	1/2 1 1/2 1
	b)	Find $\frac{dy}{dx}$ if $x^2 + 3xy + y^2 = 5$	04
	Ans	$x^2 + 3xy + y^2 = 5$ $2x + 3 \left[x \frac{dy}{dx} + y(1) \right] + 2y \frac{dy}{dx} = 0$ $2x + 3x \frac{dy}{dx} + 3y + 2y \frac{dy}{dx} = 0$ $(3x + 2y) \frac{dy}{dx} = -2x - 3y$	2 1/2 1/2

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3.	b)	$\frac{dy}{dx} = \frac{-2x-3y}{3x+2y} = \frac{-(2x+3y)}{3x+2y}$	1
	c)	Evaluate: $\int \frac{\log(\tan \frac{x}{2})}{\sin x} dx$	04
	Ans	$\int \frac{\log(\tan \frac{x}{2})}{\sin x} dx$ <p>Put $\log(\tan \frac{x}{2}) = t$</p> $\frac{1}{\tan \frac{x}{2}} \sec^2 \frac{x}{2} \left(\frac{1}{2}\right) dx = dt$ $\left(\frac{1}{2}\right) \frac{\cos \frac{x}{2}}{\sin \frac{x}{2}} \cdot \frac{1}{\cos^2 \frac{x}{2}} dx = dt$ $\therefore \frac{1}{2 \sin \frac{x}{2} \cos \frac{x}{2}} dx = dt$ $\therefore \frac{1}{\sin x} dx = dt$ $\therefore \int t dt$ $= \frac{t^2}{2} + c$ $= \frac{(\log(\tan \frac{x}{2}))^2}{2} + c$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p>
	d)	Find the equation of tangent to the circle $x^2 + y^2 + 6x - 6y - 7 = 0$ at a point it cuts the x -axis	04
	Ans	$x^2 + y^2 + 6x - 6y - 7 = 0$ <p>\therefore tangent cuts x-axis $\therefore y = 0$</p> $\therefore x^2 + (0)^2 + 6x - 6(0) - 7 = 0$ $\therefore x^2 + 6x - 7 = 0$ $\therefore x = 1 \text{ and } x = -7 \quad \therefore \text{Points are } (1,0) \text{ and } (-7,0)$ $\therefore x^2 + y^2 + 6x - 6y - 7 = 0$ $\therefore 2x + 2y \frac{dy}{dx} + 6 - 6 \frac{dy}{dx} = 0$ $\therefore (2y - 6) \frac{dy}{dx} = -2x - 6$	<p>1</p> <p>1/2</p>

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3.	d)	$\therefore \frac{dy}{dx} = \frac{-2x-6}{2y-6}$ <p>at (1,0)</p> $\text{Slope} = \frac{dy}{dx} = \frac{-2(1)-6}{2(0)-6} = \frac{-8}{-6} = \frac{4}{3}$ <p>\therefore equation is</p> $y - y_1 = m(x - x_1)$ $y - 0 = \frac{4}{3}(x - 1)$ $3y = 4x - 4$ $4x - 3y - 4 = 0$ <p>at (-7,0)</p> $\text{Slope} = \frac{dy}{dx} = \frac{-2(-7)-6}{2(0)-6} = \frac{8}{-6} = \frac{-4}{3}$ <p>\therefore equation is</p> $y - 0 = \frac{-4}{3}(x + 7)$ $3y = -4x - 28$ $4x + 3y + 28 = 0$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>
4.	a)	<p>Attempt any THREE of the following:</p> <p>Evaluate : $\int \frac{1}{5+4\cos x} dx$</p> <p>Ans</p> $\int \frac{1}{5+4\cos x} dx$ <p>Put $\tan \frac{x}{2} = t \quad \therefore \cos x = \frac{1-t^2}{1+t^2}, \quad dx = \frac{2dt}{1+t^2}$</p> $\therefore \int \frac{dx}{5+4\cos x} = \int \frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2}$ $= 2 \int \frac{1}{t^2+9} dt$ $= 2 \int \frac{1}{t^2+3^2} dt$ $= 2 \times \frac{1}{3} \tan^{-1}\left(\frac{t}{3}\right) + c$ $= \frac{2}{3} \tan^{-1}\left(\frac{\tan \frac{x}{2}}{3}\right) + c$	<p>12</p> <p>04</p> <p>1</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1/2</p>

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4.	b)	<p>Evaluate: $\int \frac{x+1}{x(x^2-4)} dx$</p> <p>Ans $\int \frac{x+1}{x(x^2-4)} dx = \int \frac{x+1}{x(x-2)(x+2)} dx$</p> <p>Let $\frac{x+1}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$</p> <p>$x+1 = A(x-2)(x+2) + Bx(x+2) + Cx(x-2)$</p> <p>put $x=0 \therefore A = \frac{-1}{4}$</p> <p>put $x=2 \therefore B = \frac{3}{8}$</p> <p>put $x=-2 \therefore C = \frac{-1}{8}$</p> <p>$\frac{x+1}{x(x-2)(x+2)} = \frac{-1}{4x} + \frac{3}{8(x-2)} + \frac{-1}{8(x+2)}$</p> <p>$\int \frac{x+1}{x(x-2)(x+2)} dx = \int \left(\frac{-1}{4x} + \frac{3}{8(x-2)} + \frac{-1}{8(x+2)} \right) dx$</p> <p>$= \frac{-1}{4} \log x + \frac{3}{8} \log(x-2) - \frac{1}{8} \log(x+2) + c$</p>	<p>04</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2+1/2+1/2</p>
	c)	<p>Evaluate: $\int \cos(\log x) dx$</p> <p>Ans $\int \cos(\log x) dx$</p> <p>Put $\log x = t \Rightarrow x = e^t$</p> <p>$\therefore \frac{1}{x} dx = dt$</p> <p>$\therefore dx = x dt$</p> <p>$\therefore dx = e^t dt$</p> <p>$\therefore \int e^t \cos t dt$</p> <p>$= \frac{e^t}{1+1} (1 \cos t + 1 \sin t) + c$</p> <p>$= \frac{x}{2} (\cos(\log x) + \sin(\log x)) + c$</p>	<p>04</p> <p>1/2</p> <p>1</p> <p>1</p> <p>1</p> <p>1/2</p>

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4.	c)	<p>OR</p> $\int \cos(\log x) dx$ <p>Put $\log x = t \Rightarrow x = e^t$</p> $\therefore \frac{1}{x} dx = dt$ $\therefore dx = x dt$ $\therefore dx = e^t dt$ $\therefore I = \int e^t \cos t dt$ $= \cos t \int e^t dt - \int \left(\int e^t dt \frac{d}{dt} \cos t \right) dt$ $= \cos t e^t - \int e^t (-\sin t) dt$ $= \cos t e^t + \int e^t \sin t dt + c$ $= \cos t e^t + e^t \sin t - \int e^t \cos t dt + c$ $\therefore I = \cos t e^t + e^t \sin t - I + c$ $\therefore 2I = \cos t e^t + e^t \sin t + c$ $\therefore I = \frac{e^t}{2} (\cos t + \sin t) + c$ $\therefore I = \frac{x}{2} (\cos(\log x) + \sin(\log x)) + c$ <p>OR</p> $I = \int \cos(\log x) dx$ $\therefore I = \int \cos(\log x) \cdot 1 dx$ $\therefore I = \cos(\log x) \int 1 dx - \int \left(\int 1 dx \frac{d}{dx} \cos(\log x) \right) dx$ $\therefore I = \cos(\log x) x - \int x \left(\frac{-\sin(\log x)}{x} \right) dx$ $\therefore I = x \cos(\log x) + \int \sin(\log x) dx$ $\therefore I = x \cos(\log x) + \int \sin(\log x) \cdot 1 dx$ $\therefore I = x \cos(\log x) + \left[\sin(\log x) x - \int x \left(\frac{\cos(\log x)}{x} \right) dx \right]$ $\therefore I = x \cos(\log x) + x \sin(\log x) - \int \cos(\log x) dx$ $\therefore I = x \cos(\log x) + x \sin(\log x) - I + c$ $\therefore 2I = x (\cos(\log x) + \sin(\log x)) + c$	<p>1</p> <p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p> <p>1</p>

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4.	c)	$\therefore I = \frac{x}{2} [\cos(\log x) + \sin(\log x)] + c$	1
	d)	Evaluate : $\int \frac{1}{x^2 + 4x + 9} dx$	04
	Ans	$\int \frac{1}{x^2 + 4x + 9} dx$ $\text{Third term} = \left(\frac{1}{2} \times 4\right)^2 = 4$ $= \int \frac{1}{x^2 + 4x + 4 - 4 + 9} dx$ $= \int \frac{1}{(x+2)^2 + (\sqrt{5})^2} dx$ $= \frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{x+2}{\sqrt{5}}\right) + c$ <p style="text-align: center;">OR</p> $\int \frac{1}{x^2 + 4x + 9} dx$ $\text{Third term} = \frac{(M.T.)^2}{4(F.T.)} = 4$ $= \int \frac{1}{x^2 + 4x + 4 - 4 + 9} dx$ $= \int \frac{1}{(x+2)^2 + (\sqrt{5})^2} dx$ $= \frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{x+2}{\sqrt{5}}\right) + c$	1 1 1 1 1 1 1 1
	e)	Evaluate $\int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+3}} dx$	04
	Ans	$\int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+3}} dx \text{-----(1)}$ $I = \int_1^5 \frac{\sqrt{9-(1+5-x)}}{\sqrt{9-(1+5-x)} + \sqrt{(1+5-x)+3}} dx$	1

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4.	e)	$\therefore I = \int_1^5 \frac{\sqrt{x+3}}{\sqrt{x+3} + \sqrt{9-x}} dx \text{----- (2)}$ <p>add (1) and (2)</p> $I + I = \int_1^5 \frac{\sqrt{9-x}}{\sqrt{9-x} + \sqrt{x+3}} dx + \int_1^5 \frac{\sqrt{x+3}}{\sqrt{x+3} + \sqrt{9-x}} dx$ $\therefore 2I = \int_1^5 \frac{\sqrt{9-x} + \sqrt{x+3}}{\sqrt{9-x} + \sqrt{x+3}} dx$ $\therefore 2I = \int_1^5 1 dx$ $\therefore 2I = [x]_1^5$ $\therefore 2I = 5 - 1$ $\therefore 2I = 4$ $I = 2$	<p>½</p> <p>½</p> <p>½</p> <p>1</p> <p>½</p>
5.	a) Ans	<p>Attempt any TWO of the following:</p> <p>Find the area of the loop of a curve $y^2 = x^2(1-x)$.</p> $y^2 = x^2(1-x)$ $y = x\sqrt{(1-x)}$ <p>at $y=0, x^2(1-x)=0$</p> $\therefore x=0, 1$ $\therefore A_1 = \int_0^1 y dx$ $= \int_0^1 x\sqrt{1-x} dx$ $= \int_0^1 (1-x)\sqrt{x} dx$ $= \int_0^1 (\sqrt{x} - x^{3/2}) dx$ $= \left[\frac{2}{3} x^{3/2} - \frac{2}{5} x^{5/2} \right]_0^1$ $= \left[\frac{2}{3} - \frac{2}{5} \right] - 0$ $= \frac{4}{15} \text{ or } 0.267$ $\therefore \text{Area of loop} = 2 \times A_1 = 2 \times \frac{4}{15} = \frac{8}{15} \text{ or } 0.533$	<p>12</p> <p>06</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>

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5.	a)	<p>OR</p> $y^2 = x^2(1-x) \quad \therefore y = x\sqrt{(1-x)}$ <p>at $y = 0, x^2(1-x) = 0$</p> $\therefore x = 0, 1$ $\therefore A = \int_0^1 y dx$ $= \int_0^1 x\sqrt{1-x} dx$ <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;"> <p>put $t = 1 - x$ $\therefore dt = -dx$ $\therefore -dt = dx$</p> </div> <div style="border: 1px solid black; padding: 5px; margin-right: 10px;"> <p>when $x \rightarrow 0$ to 1 $t \rightarrow 1$ to 0</p> </div> </div> $= -\int_1^0 (1-t)\sqrt{t} dt$ $= -\int_1^0 (\sqrt{t} - t^{3/2}) dt$ $= -\left[\frac{2}{3}t^{3/2} - \frac{2}{5}t^{5/2} \right]_1^0$ $= -\left[0 - \left(\frac{2}{3} - \frac{2}{5} \right) \right]$ $= \frac{4}{15} \text{ or } 0.267$ $\therefore \text{Area of loop} = 2 \times A_1 = 2 \times \left(\frac{4}{15} \right) = \frac{8}{15} \text{ or } 0.533$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
	b)	Attempt the following:	06
	(i)	Form the differential equation of $y = a \sin x + b \cos x$	03
	Ans	$y = a \sin x + b \cos x$ $\therefore \frac{dy}{dx} = a \cos x - b \sin x$ $\therefore \frac{d^2y}{dx^2} = -a \sin x - b \cos x$ $\therefore \frac{d^2y}{dx^2} = -(a \sin x + b \cos x)$ $\frac{d^2y}{dx^2} = -y$ $\frac{d^2y}{dx^2} + y = 0$	<p>1</p> <p>1</p> <p>1/2</p> <p>1/2</p>

OUR CENTERS :

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5.	b)(ii)	<p>Solve : $\frac{dy}{dx} + \frac{y}{x} = x^2$</p> <p>Ans $\frac{dy}{dx} + \frac{y}{x} = x^2$ Comparing with $\frac{dy}{dx} + Py = Q$</p> <p>$\therefore P = \frac{1}{x}$ and $Q = x^2$</p> <p>$IF = e^{\int \frac{1}{x} dx} = e^{\log x} = x$</p> <p>$\therefore$ Solution is $y \cdot IF = \int Q \cdot IF dx + c$</p> <p>$y \cdot x = \int x^2 \cdot x dx + c$</p> <p>$xy = \int x^3 dx + c$</p> <p>$xy = \frac{x^4}{4} + c$</p>	03
	c)	<p>A resistance of 100Ω and inductance of 0.1 henries are connected in series with a battery of 20 volts. find the current in the circuit at any instant, if the relation between L, R and E is $L \frac{di}{dt} + Ri = E$</p> <p>Ans $L \frac{di}{dt} + Ri = E$</p> <p>$\therefore \frac{di}{dt} + \frac{R}{L} i = \frac{E}{L}$ Comparing with $\frac{dy}{dx} + Py = Q$</p> <p>$\therefore P = \frac{R}{L}$ and $Q = \frac{E}{L}$</p> <p>$IF = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L} t}$</p> <p>$\therefore$ Solution is $i \cdot IF = \int Q \cdot IF dt + c$</p> <p>$i \cdot e^{\frac{R}{L} t} = \int \frac{E}{L} e^{\frac{R}{L} t} dt + c$</p> <p>$i \cdot e^{\frac{R}{L} t} = \frac{E}{L} \frac{e^{\frac{R}{L} t}}{\frac{R}{L}} + c$</p> <p>$i \cdot e^{\frac{R}{L} t} = \frac{E}{R} e^{\frac{R}{L} t} + c$</p> <p>Initially at $t = 0, i = 0 \therefore c = \frac{-E}{R}$</p> <p>$\therefore i \cdot e^{\frac{R}{L} t} = \frac{E}{R} e^{\frac{R}{L} t} + \left(\frac{-E}{R} \right)$</p>	06

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5.	c)	$i = \frac{E}{R} \left(1 - e^{-\frac{R}{L}t} \right)$ <p>When R=100 , L = 0.1 , E= 20</p> $i = \frac{20}{100} \left(1 - e^{-\frac{100}{0.1}t} \right)$ $i = 0.2 \left(1 - e^{-1000t} \right)$	1																
6.	a)(i)	<p>Attempt any TWO of the following:</p> <p>Using trapezoidal rule, evaluate $\int_0^6 f(x) dx$</p> <table border="1"> <thead> <tr> <th>x</th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> </tr> </thead> <tbody> <tr> <td>f(x)</td> <td>1</td> <td>0.5</td> <td>0.3333</td> <td>0.25</td> <td>0.2</td> <td>0.6666</td> <td>0.1428</td> </tr> </tbody> </table>	x	0	1	2	3	4	5	6	f(x)	1	0.5	0.3333	0.25	0.2	0.6666	0.1428	12
x	0	1	2	3	4	5	6												
f(x)	1	0.5	0.3333	0.25	0.2	0.6666	0.1428												
	Ans	$\int_a^b f(x) dx = \frac{h}{2} [(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})]$ <p>a = 0, b = 6 and h = 1</p> $\therefore \int_0^6 f(x) dx = \frac{1}{2} [(1 + 0.1428) + 2(0.5 + 0.3333 + 0.25 + 0.2 + 0.6666)]$ $= 2.5213$	03																
	a)(ii)	<p>Using Simpson's $\frac{1}{3}$rd rule, evaluate $\int_1^2 \frac{1}{x} dx$ given by</p> <table border="1"> <thead> <tr> <th>x</th> <th>1</th> <th>1.25</th> <th>1.5</th> <th>1.75</th> <th>2</th> </tr> </thead> <tbody> <tr> <td>y = f(x)</td> <td>1</td> <td>0.8</td> <td>0.6666</td> <td>0.5714</td> <td>0.5</td> </tr> </tbody> </table>	x	1	1.25	1.5	1.75	2	y = f(x)	1	0.8	0.6666	0.5714	0.5	03				
x	1	1.25	1.5	1.75	2														
y = f(x)	1	0.8	0.6666	0.5714	0.5														
	Ans	$\int_a^b f(x) dx = \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$ <p>Let y = f(x) = $\frac{1}{x}$ a = 1, b = 2 and h = 0.25</p> $\therefore \int_1^2 f(x) dx = \frac{0.25}{3} [(1 + 0.5) + 4(0.8 + 0.5714) + 2(0.6666)]$ $= 0.6932$	2																
			1																

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Q. No.	Sub Q. N.	Answer	Marking Scheme																
6.	b)	Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ Using Simpson's 1/3 rd rule divide the interval [0,1] into six equal parts. Find approximate value of π .	06																
	Ans	Let $y = \frac{1}{1+x^2}$ $a = 0, b = 1$ and $n = 6$ $\therefore h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6}$	1																
		<table border="1"> <tr> <td>x</td> <td>0</td> <td>$\frac{1}{6}$</td> <td>$\frac{1}{3}$</td> <td>$\frac{1}{2}$</td> <td>$\frac{2}{3}$</td> <td>$\frac{5}{6}$</td> <td>1</td> </tr> <tr> <td>$y = \frac{1}{1+x^2}$</td> <td>1</td> <td>$\frac{36}{37}$</td> <td>$\frac{9}{10}$</td> <td>$\frac{4}{5}$</td> <td>$\frac{9}{13}$</td> <td>$\frac{36}{61}$</td> <td>$\frac{1}{2}$</td> </tr> </table>	x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1	$y = \frac{1}{1+x^2}$	1	$\frac{36}{37}$	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{9}{13}$	$\frac{36}{61}$	$\frac{1}{2}$	2
x	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1												
$y = \frac{1}{1+x^2}$	1	$\frac{36}{37}$	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{9}{13}$	$\frac{36}{61}$	$\frac{1}{2}$												
		Using Simpson's 1/3 rd rule $\int_a^b f(x) dx = \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$ $\therefore \int_0^1 f(x) dx = \frac{1/6}{3} \left[\left(1 + \frac{1}{2}\right) + 4\left(\frac{36}{37} + \frac{4}{5} + \frac{36}{61}\right) + 2\left(\frac{9}{10} + \frac{9}{13}\right) \right]$ $= 0.7854$ $\therefore \int_0^1 \frac{1}{1+x^2} dx = 0.7854$ $\therefore [\tan^{-1} x]_0^1 = 0.7854$ $[\tan^{-1}(1)] - [\tan^{-1}(0)] = 0.7854$ $\frac{\pi}{4} = 0.7854$ $\pi = 3.142$	1 1																
		OR Let $y = \frac{1}{1+x^2}$ $a = 0, b = 1$ and $n = 6$ $\therefore h = \frac{b-a}{n} = \frac{1-0}{6} = \frac{1}{6} = 0.1667$	1/2																
		<table border="1"> <tr> <td>x</td> <td>0</td> <td>0.1667</td> <td>0.3334</td> <td>0.5001</td> <td>0.6668</td> <td>0.8335</td> <td>1</td> </tr> <tr> <td>$y = \frac{1}{1+x^2}$</td> <td>1</td> <td>0.9730</td> <td>0.9</td> <td>0.8</td> <td>0.6922</td> <td>0.5901</td> <td>0.5</td> </tr> </table>	x	0	0.1667	0.3334	0.5001	0.6668	0.8335	1	$y = \frac{1}{1+x^2}$	1	0.9730	0.9	0.8	0.6922	0.5901	0.5	2
x	0	0.1667	0.3334	0.5001	0.6668	0.8335	1												
$y = \frac{1}{1+x^2}$	1	0.9730	0.9	0.8	0.6922	0.5901	0.5												
		Using Simpson's 1/3 rd rule $\int_a^b f(x) dx = \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})]$																	

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6.	b)	$\therefore \int_0^1 f(x) dx = \frac{0.1667}{3} [(1+0.5) + 4(0.9730 + 0.8 + 0.5901) + 2(0.9 + 0.6922)]$ $= 0.7855$ $\int_0^1 \frac{1}{1+x^2} dx = 0.7855$ $[\tan^{-1} x]_0^1 = 0.7855$ $[\tan^{-1}(1)] - [\tan^{-1}(0)] = 0.7855$ $\frac{\pi}{4} = 0.7855$ $\pi = 3.142$	<p>1</p> <p>1</p> <p>½</p> <p>½</p>															
	c)	<p>Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ Using Simpson's 3/8th rule.</p> <p>Ans Consider $n = 6$</p> $y = \frac{1}{1+x^2} \quad a = 0, \quad b = 6$ $\therefore h = \frac{b-a}{n} = \frac{6-0}{6} = 1$ <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>x</th> <th>0</th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> </tr> </thead> <tbody> <tr> <td>$y = \frac{1}{1+x^2}$</td> <td>1</td> <td>0.5</td> <td>0.2</td> <td>0.1</td> <td>0.0588</td> <td>0.0385</td> <td>0.0270</td> </tr> </tbody> </table> <p>Using Simpson's 3/8th rule.</p> $\int_a^b f(x) dx = \frac{3h}{8} [(y_0 + y_n) + 3(y_1 + y_2 + y_4 + \dots + y_{n-1}) + 2(y_3 + y_6 + \dots + y_{n-3})]$ $\therefore \int_0^6 \frac{1}{1+x^2} dx = \frac{3(1)}{8} [(1+0.0270) + 3(0.5+0.2+0.0588+0.0385) + 2(0.1)]$ $\therefore \int_0^6 \frac{1}{1+x^2} dx = 1.3571$ <p><u>Note:</u> If the student has considered any value of n and attempted to solve give appropriate marks.</p>	x	0	1	2	3	4	5	6	$y = \frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.0588	0.0385	0.0270
x	0	1	2	3	4	5	6											
$y = \frac{1}{1+x^2}$	1	0.5	0.2	0.1	0.0588	0.0385	0.0270											



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		<p><u>Important Note</u></p> <p><i>In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.</i></p>	

Pinnacle